A Text Book on

ENGINEERING MECHANICS

for

GATE

PSUs & Other Competitive Exams

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Preface

At the outset I must thank Mr. B. Singh (Chairman and Managing Director of MADE EASY Group) who has very kindly given me an opportunity to serve the student community through the publication of this book on Engineering Mechanics.

The present book has been designed as a self study book taking into account the severe shortage of technical teachers in engineering colleges and technical institutions.

The text in the book is well explained through examples supplemented by self explanatory illustrations, exercises supplemented by hints, key points to remember, thought provoking multiple choice questions, special problems so that a student can learn this basic subject in the shortest possible time.

The book covers all the syllabi in Engineering Mechanics of GATE, PSUs, all the universities, IITs, NITs, deemed universities. Students appearing in competitive examinations and other competitive examinations will find the book as an asset to them. The book also serves the purpose of AMIE students.

The book will greatly help the students who could not grasp the subject in the class room.

Any suggestion for the improvement in the text of the book will be thankfully acknowledged.

Dr. U. C. Jindal
Author

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01
CHAPTER

Vector Quantities in Mechanics

1.1 Introduction

Study of engineering mechanics is incomplete if the vector quantities are not understood thoroughly for the correct solution of any problem in engineering mechanics *specially three-dimensional problems.*

Vector quantities as position vector, displacement vector, force vector, moment of a force about a point or about an axis, couple vector, addition and subtraction of couple vectors form the text of this chapter.

1.2 Position Vector

A position vector of a point is defined by the *position of a point in any coordinate system*. A vector starting from the origin of a coordinate system to the point in space is termed as *position vector.*

Fig.1.1 (*a*) shows a point P in x-y coordinate system with coordinates $P(x_1, y_1)$. Position vector of P is OP starting from the origin of coordinate system. Position vector R_1 from O to P can be expressed as

$$R_1 = x_1 i + y_1 j$$

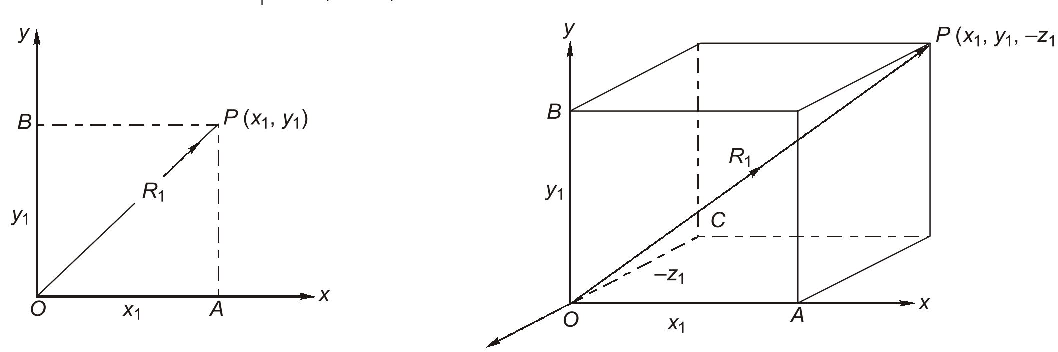

Fig. 1.1

Similarly Fig. 1.1 (*b*) shows a point P in x-y-z coordinate system with coordinates $P(x_1, y_1, -z_1)$, $\overrightarrow{OP}$ is the position vector of point P and can be expressed as

$$R_1 = x_1 i + y_1 j - z_1 k.$$

Example 1.1 (*a*) Mark the position vector of a point $P(4, -3)$, (*b*) Show the position vector of a point $P(4, -3, +2)$. What are its direction cosines?

Solution (a) Take x-y coordinates system as shown in **Fig. 1.2**. Take OA = +4 units on some scale, and AP = –3 units (in the negative direction of y-axis). Vector, R is position vector of point P, *i.e.*,

$$R = 4i - 3j$$

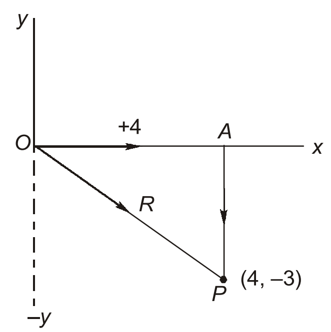

Fig. 1.2

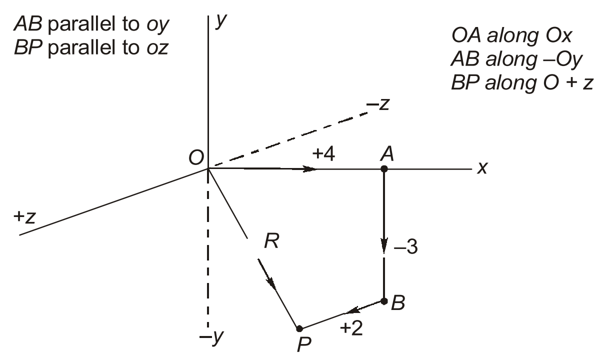

Fig. 1.3

(b) Consider x-y-z coordinate axes as shown in **Fig. 1.3**. To reach the point P(4, –3, 2), take OA = +4 units along positive x-axis, from A to B, take –3 units along direction of the negative y-axis as shown in Fig. 1.3. From B draw a line parallel to z-axis and take BP = +2 units on same scale. Vector from O to P, is a position vector of point P,

i.e., $$R = 4i - 3j + 2k$$

Magnitude of position vector

$$|R| = \sqrt{4^2 + (-3)^2 + 2^2} = \sqrt{29} = 5.385 \text{ units}$$

Direction cosines $$l = \frac{4}{5.385} = +0.743, \quad m = \frac{-3}{5.385} = -0.557$$

$$n = \frac{2}{5.385} = +0.371$$

Exercise 1.1 (a) Show the position vector of a point P(–4, +6) in x-y coordinate system.

[**Ans:** $R = -4i + 6j$, show the position vector in x-y plane].

(b) Show the position of vector a point P(–6, +4, –2) in x-y-z coordinate system. What is the magnitude of position vector and what are its direction cosines?

[**Ans:** Show the position vector on x-y-z coordinate system, $|R| = 7.483$ units

direction cosines; $l = -0.801$, $m = +0.534$, $n = -0.267$].

1.3 Displacement Vector

Displacement vector between two points P and Q in space is defined as $\vec{S}_{PQ}$, displacement from point P to point Q (**Fig. 1.4**).

Point P and Q in space have coordinates $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

Then displacement vector $$S_{PQ} = (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1) k$$

$$S_{PQ} = R_2 - R_1$$

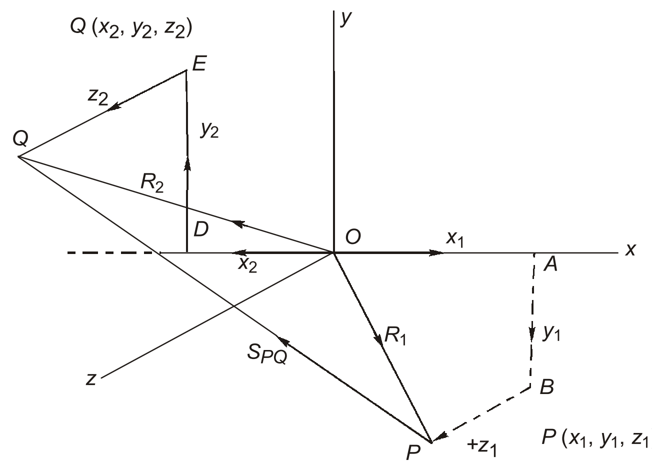

Fig. 1.4

$$= (x_2\, i + y_2\, j + z_2\, k) - (x_1\, i + y_1\, j + z_1\, k)$$

$$= (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1)\, k$$

Please note that arrow of displacement vector S_{PQ}, is pointing towards Q from point P.

Example 1.2 (*a*) What is the displacement vector from P (6, –4) m to Q (–3, 6) m? What are direction cosines of this vector? What is unit vector along displacement vector S_{PQ}?

Solution (*a*) Points P (6, –4) m and Q (–3, 6) m have position vectors as follows

$$R_1 = 6i - 4j \text{ m}, \quad R_2 = -3i + 6j \text{ m}$$

Displacement vector, $S_{PQ} = R_2 - R_1 = (-3i + 6j) - (6i - 4j)$ m

$$= -9i + 10j \text{ m}$$

$$|S_{PQ}| = \sqrt{(-9)^2 + (10)^2} = 13.453 \text{ m}$$

Direction cosines, $l = \dfrac{-9}{13.453} = -0.668, \quad m = \dfrac{+10}{13.453} = +0.743$

Unit vector along S_{PQ}, $\bar{r} = -0.668i + 0.743j$

Exercise 1.2 (*a*) What is the displacement vector between points A (4, –6) and B (5, 3) m? What is unit vector along displacement vector S_{AB}?

[**Ans:** $1i + 9j$; $\bar{r} = 0.11i + 0.994j$].

1.4 Force Vector

A force vector is represented by its rectangular components F_x, F_y, F_z along x, y, z cartesian coordinates as shown in Fig. 1.5.

$$F = F_x\, i + F_y\, j + F_z\, k$$

Magnitude of force vector

$$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Direction cosines

$$\cos\alpha = \frac{F_x}{|F|} = l$$

$$\cos\beta = \frac{F_y}{|F|} = m$$

$$\cos\gamma = \frac{F_z}{|F|} = n,$$

where α, β and γ are the angles between the force vector F and coordinate axes x, y and z respectively.

Moreover $l^2 + m^2 + n^2 = 1$.

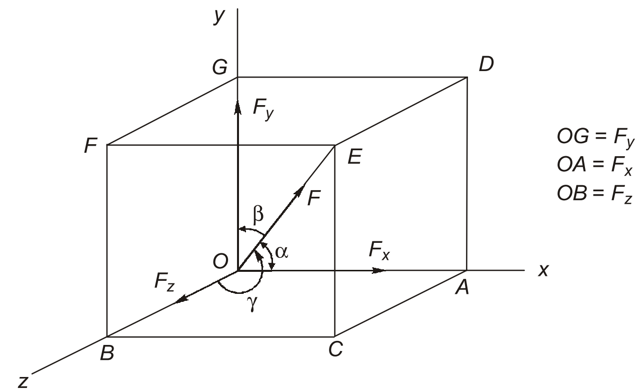

Fig. 1.5

Example 1.3 A force is, $F = 40i - 50j + 35k$ Newton. What is magnitude of F and what are direction cosines? Show force vector F in cartesian coordinate system.

Solution Fig. 1.6 shows force vector F

$= 40i - 50j + 35k$

to some suitable scale

$$F = \sqrt{40^2 + (-50)^2 + 35^2}$$

$$= \sqrt{1600 + 2500 + 1225}$$

$$= 72.97 \text{ N}$$

Direction cosines

$$l = \cos\alpha = \frac{40}{72.97} = +0.548$$

$$m = \cos\beta = -\frac{50}{72.97} = -0.685$$

$$n = \cos\gamma = \frac{35}{72.97} = +0.48$$

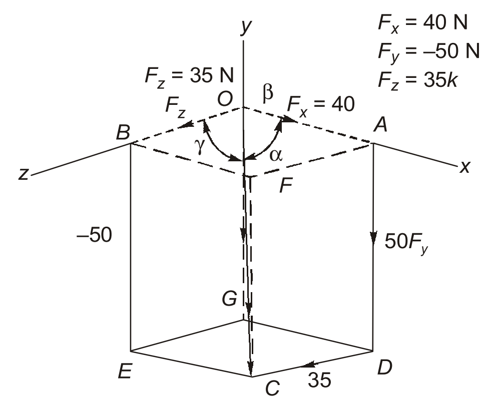

Fig. 1.6

Exercise 1.3: Show a force vector $-40i + 60j - 30k$ along cartesian coordinates. What are its magnitude and direction cosines?

[**Ans:** 78.10 N, –0.512, +0.768, –0.384].

1.5 Moment of a Force about a Point

Moment of a force vector F about point O, is a moment vector M as shown in the Fig. 1.7.

Moment = force × perpendicular distance from point O on force F

$= d \times F$

d = perpendicular distance from O on the line of force vector F = (OA)

Physically, moment M represents the tendency of the force F to rotate the body (on which force acts) about an axis which is passing through O, and this axis is perpendicular to the plane

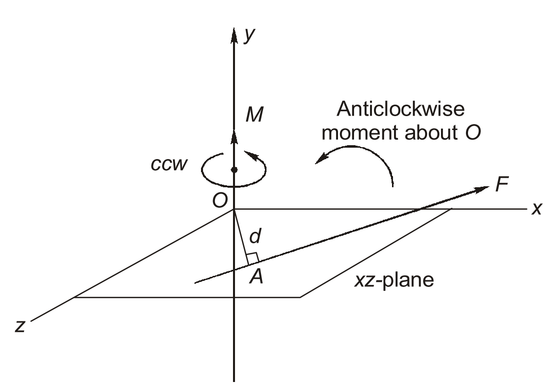

Fig. 1.7

of force F. In the **Fig. 1.7** xz is the plane of force F and M vector is perpendicular to xz plane *i.e.*, in the direction of y-axis. *If moment is anticlockwise, then moment vector is positive*, *i.e.*, it is represented along positive direction of y-axis as shown in **Fig. 1.7.**

(Using the right hand rule, the direction of vector M is determined).

Another approach to determine moment vector M, is to use a position vector r of any point P on the line of action of force F. Then moment vector M is cross product of r and F *i.e.*, $r \times F$.

$$M = r \times F = \text{position vector from } O \times \text{force vector } F$$

$$\text{Magnitude of } |M| = |r||F|\sin\beta = |F||r|\sin\alpha$$

$$= F \,.\, d$$

as shown in **Fig. 1.8**. In the **Fig. 1.8**, OP is position vector of any point P from the origin, on the force vector F.

In this case $|r|\sin\alpha = d$, perpendicular distance from origin O to the line of action of F.

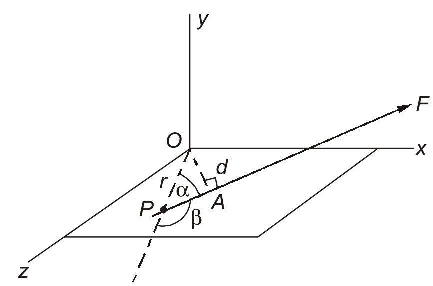

Fig. 1.8

Example 1.4 A cantilever beam OAC, is fixed in wall at left end O. A force of 80 N inclined at an angle of 45° as shown is applied on the beam. If $OA = 2$ m, what is the moment of the force about point O. Verify this value of moment by taking position vector OA and force vector F in terms of unit vectors **(Fig. 1.9).**

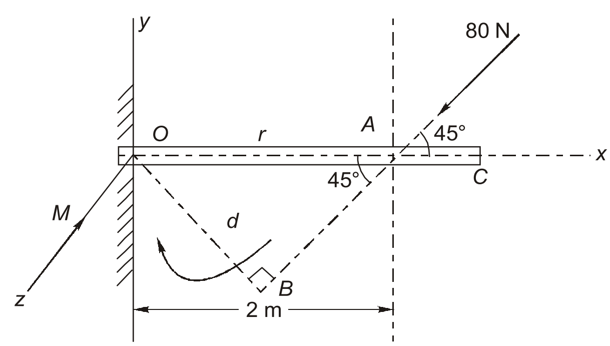

Fig. 1.9

Solution Line of action of force F is extended as shown. A perpendicular from O on the extended line of action of force is drawn.

Perpendicular distance

$$d = OA \sin 45^\circ$$
$$= 2 \times 0.707 = 1.414 \text{ m}$$

Moment of the force about O,

$$M = 80 \times 1.414 \text{ Nm (clockwise)}$$
$$= 113.12 \text{ Nm (clockwise)}$$
$$= -113.12 \text{ Nm}$$

Exercise 1.4 A cantilever OA, 3 m long is fixed at end O. A vertical load of 100 N acts at end A. Determine moment of force about point O. Choose OA as position vector from O, **Fig. 1.10**, and verify the answer by vector method.

[**Ans:** 300 Nm (clockwise); $r = 3i$ m, $F = -100j$ N, $M = -300k$ Nm about z-axis].

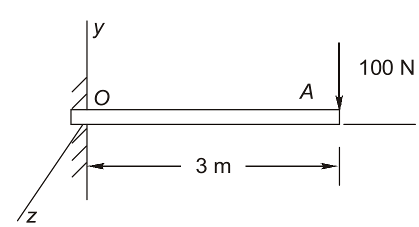

Fig. 1.10 (Ex. 1.4)

1.6 Varignon's Theorem

Consider a system of n concurrent forces $F_1, F_2, F_3, F_4, \ldots, F_n$ acting at a point P in x-y-z co-ordinate system. From O to P is the position vector of the point P. All the forces $F_1, F_2, \ldots, F_n$ are passing through the point P **(Fig. 1.11)**.

Moment of the n forces about the origin O,

$$M_0 = r \times F_1 + r \times F_2 + r \times F_3 + r \times F_4, \ldots, r \times F_n$$

$$= r \times [F_1 + F_2 + F_3 + F_4, ..., F_n]$$
$$= r \times F_R$$

where F_R is the resultant of n forces F_1, F_2, ..., F_n.

It can be concluded that the *sum of the moments about a point* of a system of concurrent forces is the same as the moment of resultant of all these forces about the same point as

$$F_R = F_1 + F_2 + F_3 + F_4, ..., F_n.$$

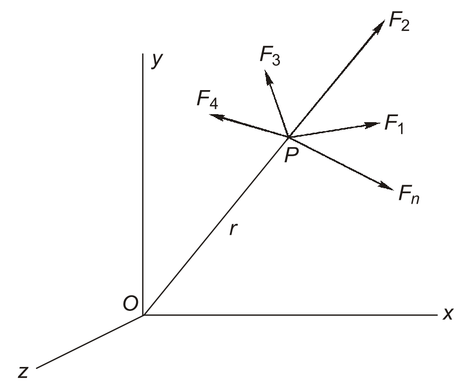

Fig. 1.11

1.6.1 Graphically

Consider a force F acting at point P of a body. Forces F_1 and F_2 represent two components of the force about any two directions (Fig. 1.12).

Moment of force F about a point O,

$$M_0 = r \times F, \quad \text{but } F = F_1 + F_2 \text{ (vectorial sum)}$$

$$M_0 = r \times (F_1 + F_2) = r \times F_1 + r \times F_2$$

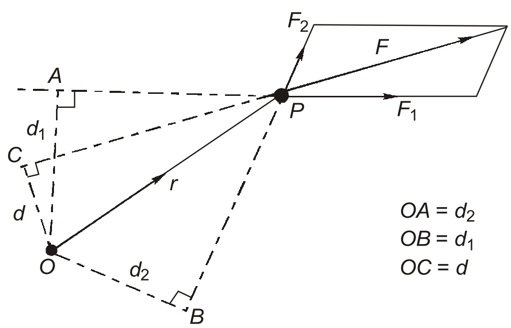

Fig. 1.12

Lines of forces F_1, F_2 and F are extended and perpendiculars drawn on these lines from the point O,

i.e., d_1 is perpendicular from O to line of action of F_1

d_2 is perpendicular from O to line of action of F_2

d is perpendicular from O to line of action of F

Taking anticlockwise moments as positive and clockwise moments as negative

$$M_0 = -F_1 d_1 + F_2 d_2 = F.d$$
$$F.d = F_2 d_2 - F_1 d_1$$

this is the *scalar equivalent* of the vector expression of M_0.

Example 1.5 Three forces F_1, F_2 and F_3 act on point P in x-y plane, as shown in Fig. 1.13. These are concurrent forces, find the resultant of the forces and resultant moment of the forces about origin O. Distance $OP = 3$ m.

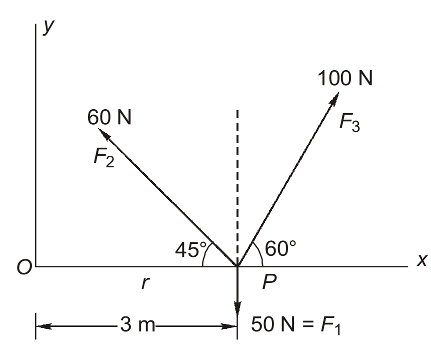

Fig. 1.13

Solution Taking clockwise moments negative and anticlock-wise moments positive. Remember that horizontal components of forces (in this problem) pass through O and will not produce any moment. So

$$M_0 = F_2 \sin 45° \times 3 + F_3 \sin 60° \times 3 - F_1 \times 3$$
$$= 3\,[60 \times \sin 45° + 100 \sin 60° - 50]$$
$$= 3\,[42.42 + 86.6 - 50] = 237.06 \text{ Nm}$$

(a positive anticlockwise moment)

Exercise 1.5 Three forces of 120 N, 80 N and 150 N act at point P, in x-y plane as shown in Fig. 1.14. What is the moment of these forces about origin O if $OP = 5$ m?

[**Ans:** –249.5 Nm, or –249.5k Nm along z-axis].

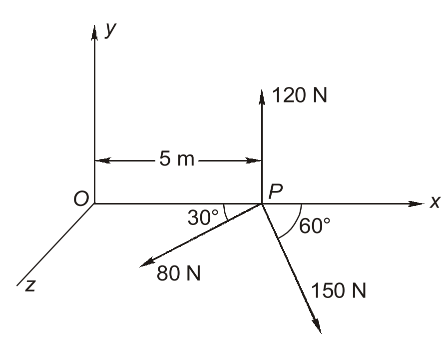

Fig. 1.14

PROBLEMS

1.1 A particle moves along a circular path of radius R in x-y plane as shown in Fig. 1.26. What is the position vector R of this particle in terms of y-co-ordinate?

Solution:
$$R^2 = x^2 + y^2$$
$$x^2 = R^2 - y^2$$
$$x = \sqrt{R^2 - y^2}$$

Position vector $R = xi + yj = \sqrt{R^2 - y^2}\,.i + yj.$

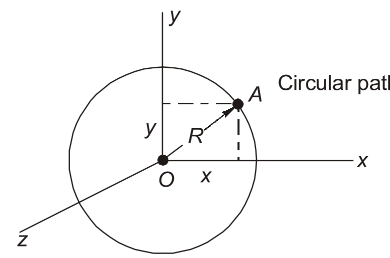

Fig. 1.26

1.2 A particle moves along a parabolic path in xy plane. If the particle has at one point a position vector $R = 4i + 3j$, give the position vector at any point on the path as a function of x-co-ordinate.

Solution: Equation of a parabola is
$$y^2 = cx$$
Fig.1.27 shows a parabola $y^2 = cx$

Putting the value of y and x $3^2 = c \times x = 4c$, as $x = 4$
$$c\text{, constant} = \frac{9}{4} = 2.25$$
$$y = \sqrt{cx} = 1.5\sqrt{x}$$
Therefore, position vector of path, $R = xi + 1.5\sqrt{x}\,j.$

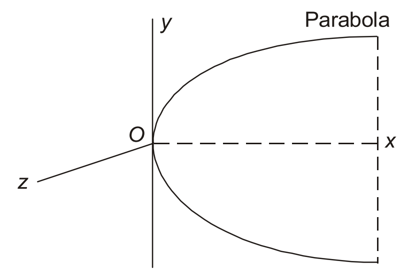

Fig. 1.27

1.3 The boom of a crane is 12 m long, it is hinged at end A. A load of 20 kN is being lifted by the crane with the help of a wire rope. What is the moment of the force about end A. Express moment as vector (Fig. 1.28)?

Solution: Force $= 20$ kN $\downarrow$

Perpendicular distance from A to line of action of force,
$$d = 12 \cos 60° = 6 \text{ m}$$
Moment of the force about A,
$$M = 6 \times 20 = 120 \text{ kNm (cw) (a negative moment).}$$
$$= -120 \text{ kNm}$$

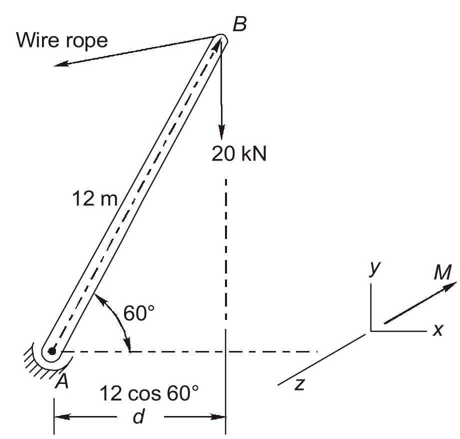

Fig. 1.28

Remember

- Vector from origin of a co-ordinate system to the point (x, y, z) is termed as position vector,
$$R = xi + yj + zk.$$
- Displacement vector between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is
$$S_{PQ} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k.$$
- Force vector, $F = F_x.i + F_y.j + F_z.k$
- Moment of a force about a point A,
$$M_A = r \times F$$
= cross product of position vector and force vector
= Magnitude of force × perpendicular distance from the point A to the line of action of the force.

- Varignon's theorem—algebraic sum of the moments of any number of concurrent forces about a point in their plane is equal to the moment of their resultant about the same point.
- Moment of a force about an axis is a scalar component, is equal to the component of the moment along that axis.
- A couple is formed by any two equal and opposite forces
  = Force × perpendicular distance between the forces in the plane of the couple.
- Couple Vector lies in a direction perpendicular to the plane of the couple.
- Component of a couple about any line $= C.\bar{r}$ dot product of couple moment and unit vector along the line.
- Couples can be added vectorially.

PRACTICE PROBLEMS

1.1 A force $F = 6i + 3j - 6k$ N acts at position (5, 3, 4) m relative to a co-ordinate systems. What is the moment of the force about the origin?

[**Ans:** $M = -30i + 54j - 3k$ Nm].

1.2 A bracket is fixed on a vertical column, 6 m high from the ground. Bracket supports a load of 15 kN at its end, 2 m from axis of column as shown in **Fig. 1.29**. What is moment of the force at the fixed end *B* of column? Express the moment as vector.

[**Ans:** $-30k$ kNm].

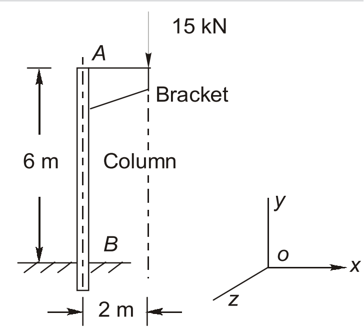

Fig. 1.29

MULTIPLE CHOICE QUESTIONS

1.1 A line is equally inclined with 3 *x*, *y*, *z*-axes. What is its direction cosine with each axis?

(*a*) 0.333 (*b*) 0.52
(*c*) 0.5777 (*d*) 0.866.

1.2 What is displacement vector of a line passing from *S* (3, 0, –4) to *P* (0, 4, 3)?

(*a*) $3i - 4j - 7k$ (*b*) $-3i + 4j + 7k$
(*c*) $-i - 3j + 4k$ (*d*) None of these.

1.3 A force $F = +5j$ kN is passing through a point (4, 0, 0), what is the moment of the force?

(*a*) $+20k$ (*b*) $-20k$
(*c*) $20j$ (*d*) None of these.

1.4 A force $P = 4i - 3j$ kN is passing through a point (4, 0, 0) m in *xyz* coordinate. What is moment of the force about the origin?

(*a*) $+12k$ kNm (*b*) $+16k$ kNm
(*c*) $-12k$ kNm (*d*) None of these.

1.5 Three concurrent forces are passing through a point *P* on *x*-axis as shown in **Fig. 1.30**, what is moment of these forces about the origin *O*?

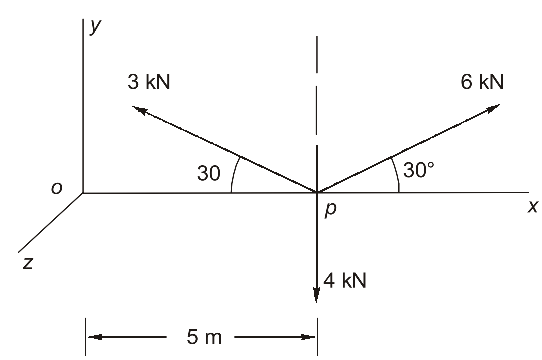

Fig. 1.30

(*a*) +22.5 kNm (*b*) +2.5 kNm
(*c*) –20 kNm (*d*) None of these

1.6 Which is a correct statement?

(*a*) Couple is a fixed vector
(*b*) Moment of couple changes with point in space
(*c*) Couple is a free vector
(*d*) All of the above

1.7 Two forces +*P* and –*P* act along two edges of a cube of side *a*. What is moment of the couple **(Fig. 1.31)?**

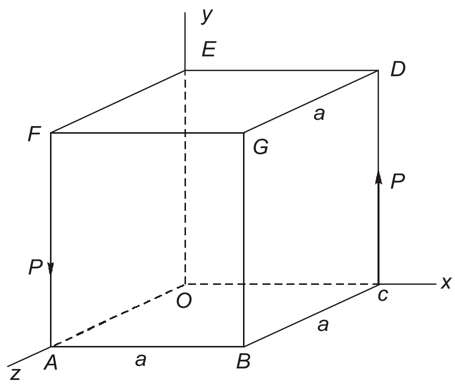

Fig. 1.31

(*a*) $+Pa$ (*b*) $+\sqrt{2}Pa$

(*c*) $-\sqrt{2}Pa$ (*d*) $+2Pa.$

1.8 What is the unit vector of the resultant of the following forces

$\bar{F}_1 = 2\hat{i}+3\hat{j}+4\hat{k}$, $\bar{F}_2 = 4\hat{i}+3\hat{j}+2\hat{k}$

(*a*) $6\hat{i}+6\hat{j}+6\hat{k}$ (*b*) $\dfrac{\hat{i}}{\sqrt{3}}+\dfrac{\hat{j}}{\sqrt{3}}+\dfrac{\hat{k}}{\sqrt{3}}$

(*c*) $-2\hat{i}+2\hat{k}$ (*d*) $2i-2\hat{k}$

[CSE, Prelims, CE, 2009]

1.9 What is the magnitude of the resultant of forces

$\vec{A} = 2\hat{i}+5\hat{j}$, $\vec{B} = 6\hat{i}-7\hat{k}$, $\vec{C} = 2\hat{i}-6\hat{j}+10k$

(*a*) 10.5 (*b*) 10

(*c*) 11.5 (*d*) 12.5

[CSE, Prelims, CE, 2008]

1.10 A force $\vec{F} = 2i+5j-k$ is passing through the origin, what is the moment about point (1, 1, 0)

(*a*) $i-j-k$ (*b*) $-i+j+k$

(*c*) $i+2k$ (*d*) $i++2j-3k$

[CSE, Prelims, CE, 2010]

1.11 A and B are the end points of a diameter of a disc rolling along a straight line with a counter clockwise angular velocity as shown in **Fig. 1.32**. Referring to the velocity vectors $\bar{V}_A$ and $\bar{V}_B$ shown in figure.

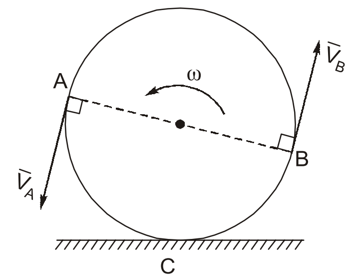

Fig. 1.32

(*a*) $\bar{V}_A$ and $\bar{V}_B$ both are correct

(*b*) $\bar{V}_A$ is correct but $\bar{V}_B$ is incorrect

(*c*) $\bar{V}_A$ and $\bar{V}_B$ both are incorrect

(*d*) $\bar{V}_A$ is correct but $\bar{V}_B$ is incorrect

[GATE : 1990, 1 Marks]

Answers

1.1 (c)	1.2 (b)	1.3 (a)	1.4 (c)	1.5 (b)
1.6 (c)	1.7 (b)	1.8 (b)	1.9 (a)	1.10 (b)
1.11 (c)				

EXPLANATIONS

1.1 (c)

$\sqrt{0.33} = 0.577.$

1.2 (b)

$(0-3)i + (4-0)\,j + (3+4)\,k = -3i + 4j + 7k.$

1.3 (a)

$F = +4j$ and point (4, 0, 0) in x, y plane moment = $+20k$ (anticlockwise) **(Fig. 1.33)**.

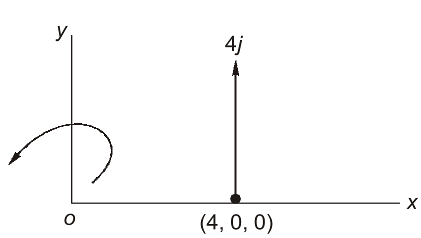

Fig. 1.33

1.4 (c)

$M_0 = -12k$ kNm **(Fig. 1.34)**.

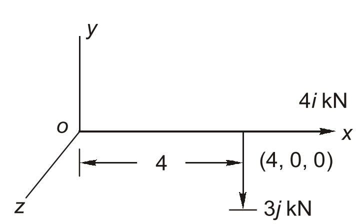

Fig. 1.34

1.5 (b)

Net force in y-direction = 3 sin 30° + 6 sin 30° – 4

= 0.5 kN

$M_0 = +5\times 0.5$ kNm (ccw) = +2.5 kNm (ccw).

1.6 (c)

Couple is a free vector **(Fig. 1.35)**

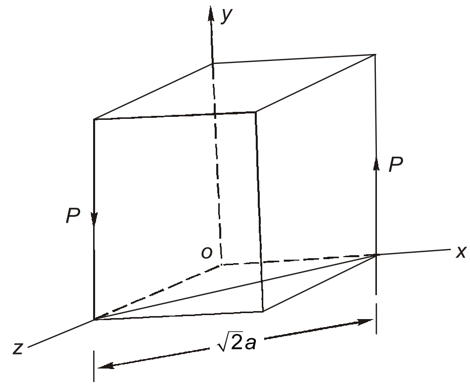

Fig. 1.35

$$\text{arm} = \sqrt{2}a$$

$$M = \sqrt{2}Pa.$$

1.7 (b)

$$\text{arm} = \sqrt{a^2 + a^2} = \sqrt{2}.a$$

$$\text{couple} = \sqrt{2}\,Pa \quad \text{(anticlockwise)}$$

1.8 (b)

$$\bar{F}_R = 6\hat{i} + 6\hat{j} + 6\hat{k}$$

$$\text{Unit vector} = \frac{6\hat{i}}{\sqrt{108}} + \frac{6\hat{j}}{\sqrt{108}} + \frac{6\hat{k}}{\sqrt{108}} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$$

1.9 (a)

$$\bar{R} = 10\hat{i} - \hat{j} + 3\hat{k}$$

$$\text{Magnitude} = \sqrt{10^2 + (-1)^2 + 3^2}$$

$$= \sqrt{100 + 1 + 9} \simeq 10.5$$

1.10 (b)

$$M_x = yF_z - zF_y$$

$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$

where, $x = 1$, $y = 1$, $z = 0$

$F_x = 2$, $F_y = 3$, $F_z = -1$

$M_x = -1$, $M_y = +1$, $M_z = 3 - 2 = 1$

$M = -i + j + k$

1.11 (c)

$\bar{V}_A$ and $\bar{V}_B$, both are incorrect

Disc rotating about point of contact C

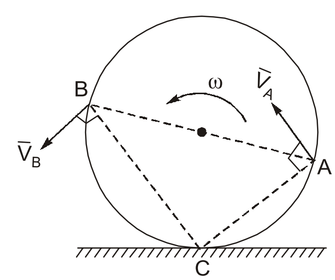

Fig. 1.36

■■■■